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LETTER TO THE EDITOR

Dynamic scaling of the interface in two-phase viscous flows in porous media

Viktor K Horváth†, Fereydoon Family and Tamás Vicsek†
Department of Physics, Emory University, Atlanta, GA 30322, USA

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Abstract. The time evolution and geometry of rough interfaces observed in experiments on immiscible displacement of viscous fluids in porous media are analysed using the concepts of dynamic scaling and self-affine fractal geometry. We find that the development of the interface is governed by dynamic scaling and we have determined the corresponding surface exponents α and β. The values of the exponents calculated for the experimental patterns are different from those obtained for a variety of two-dimensional models of marginally stable interfaces.

The far-from-equilibrium growth of surfaces [1–3] typically leads to complex patterns described by fractal geometry. If the conditions of the growth process are such that the development of the interface is neither stable nor unstable and the fluctuations are relevant, the resulting structure can be well approximated by a single valued, self-affine function [4]. This type of growth occurs in a wide range of processes, including crystal growth, thin film growth by vapour deposition, sedimentation and settling of granular materials, fluid displacement in porous media, shock waves and flame fronts, and the formation of biological patterns. Many industrial processes, such as coating, painting and soldering, also produce rough interfaces [1].

The time dependence of the surface geometry represents a natural and particularly important aspect of this kind of growth phenomena [2, 3], since the process usually starts with a simple configuration, which evolves in time into a complex shape. Consequently, many recent investigations [3] have concentrated on the dynamic scaling [5] properties of surfaces obtained in various aggregation models, [5–10] and both analytical [11, 12] and numerical solutions [13] of the related equations [14] have been carried out. As a result of these studies, our knowledge and understanding of the model systems and their theoretical description has gone through a spectacular development during the past few years [2, 3].

Much less is known about the experimental applicability of the ideas mentioned above. Only a few works have reported experimental results on the fractal properties of self-affine interfaces, [1, 15–24] and no experiment discussing the dynamic scaling [5] aspect of interface growth has been published yet.

In this letter we present the results of an experimental study of the evolution of a fluctuating rough interface. We have investigated the geometry of the interfaces in quasi-two-dimensional displacement of viscous fluids in inhomogeneous media. The evolution of the interface is found to be governed by dynamic scaling, and we have

† Permanent address: Institute for Technical Physics, HAS, Budapest, PO Box 76, 1325 Hungary.

determined the relevant surface exponents α and β [5]. We find that α and β do not have the values 1/2 and 1/3, respectively, obtained for two-dimensional numerical models of growing marginally stable interfaces, but they are consistent with the relation [6, 24, 25] $\alpha+\alpha/\beta=2$, within the experimental errors.

A growing interface can be described by the single-valued function $h(x, t)$, which gives the height of the interface at position x as a function of time $t \simeq \bar{h}$, where $\bar{h}$ is the mean height of the interface. To incorporate the scaling behaviours in both time and space one assumes that the actual geometry of the surface (in $d=2$) is described by the scaling form

$$\tilde{h}(x, t) \sim t^{\beta} f(x/t^{\beta/\alpha}) \tag{1}$$

where $\tilde{h}=h-\bar{h}$, L is the linear extension of the interface and the scaling function $f(y)$ is a fluctuating function such that $|f(y)| <$ constant for $y \gg 1$ and $f(y) \sim L^{\alpha} f(Ly)$ for $y \ll 1$. If the behaviour of the surface is given by (1), then its width $w(L, t) \equiv \langle \tilde{h}^2(x, t)\rangle^{1/2}$ scales as [5] $w(L, t) \sim t^{\beta} g(t/L^{\alpha/\beta})$. Alternatively, the height-height correlation function

$$c(x, t) = \langle [\tilde{h}(x', t') - \tilde{h}(x'+x, t'+t)]^2 \rangle_{x', t'} \tag{2}$$

for $x \ll L$ and fixed t scales as $c(x, 0) \sim x^{2\alpha}$, while for fixed x and short times $c(0, t) \sim t^{2\beta}$. Therefore, the exponents α and β can be determined from the slopes of the corresponding log-log plots of $c(x, t)$.

The experimental set-up was a linear Hele-Shaw cell made of parallel plexiglass plates of size 24×100 cm. To produce a porous medium we packed 220 μm diameter glass beads between the plates. The beads were spread randomly and homogeneously in one layer and glued to the lower plate. The upper plate was placed directly on the beads and iron rods and clamps were used to prevent the lifting of the plates. Coloured glycerol with 4 vol % of water was injected at a fixed flow rate into air between the plates along a line at a shorter sidewall. The viscosity of the glycerol was ~180 cP and the air-liquid surface tension was about ~65 dyn cm^{-1}.

In such a system the motion of the fluid-air interface is influenced by several factors, including the porosity of the medium and the type and the flow rate of the fluids [19, 23]. If, as in our experiments wetting fluid is injected slowly into a network of small pores, the meniscus instability inside a pore leads to an almost complete filling of the network behind the front [23]. Because of the applied flow rate the stabilizing effect of the pressure distribution in the glycerol can be neglected. Finally, the roughness of the interface is built up due to the random distribution of voids between the glass beads. Consequently, our experiment can be considered as a realization of marginally stable growth in the presence of fluctuations.

The evolution of the developed interface was recorded on a videotape. The video camera was fixed to a holder so that the vertical direction in the monitor was the same as the direction of the larger extension of the cell. The camera was focused to the central 18 cm of the horizontal cross section of the cell to avoid edge effects. If the interface reached a predefined height in the monitor, the cell was moved backward by 4 cm to keep the surface in the visual field of the camera.

We digitized the recorded interfaces with 768×620 spatial resolution and 1 bit of grey scale. Due to the high quality of the objective and the moving mechanism of the cell, the accuracy of the calibrated system (including the noise of the video signal) was better than 3 pixels.

We analysed the data from the moment glycerol entered the plates along a shorter side until the average height of the interface reached 3/4 of the cell size. The sampling rate was 0.28 s. The computer scanned the digitized pictures in order to get the quantized surface $h(x, t)$, which was defined as the set of points along the highest position in each column of the array of occupied pixels. Each digitized image was saved as an array of height values and this process was continued until all images in a run were recorded. Extra pixels, equivalent to 4 cm, were added to the height every time the cell was moved backward. Using this technique a digitized version of one run contained several thousand height arrays.

We used the average height $\bar{h} = \langle h(x, t) \rangle_t$ as the timescale in the calculations, because $\bar{h}$ is proportional to the time due to the constant flow velocity and the absence of holes in the region occupied by the more viscous fluid. In contrast to the results of computer simulations, in our case the data are not equally spaced in time. To account for this we introduced a parameter Δt, which defined a time range between pairs of $h(x', t')$. This parameter was 0.3 s. From the data, we determined the height–height correlation function using (2).

Figure 1 shows the digitized image of the meniscus as it evolves. The shift between the patterns is proportional to the time lapse between the pictures. The roughening of the interface is manifested by the appearance of large mountains and valleys. The three-dimensional appearance of the patterns is obtained by making the brightness of the surface at a given point proportional to its height.

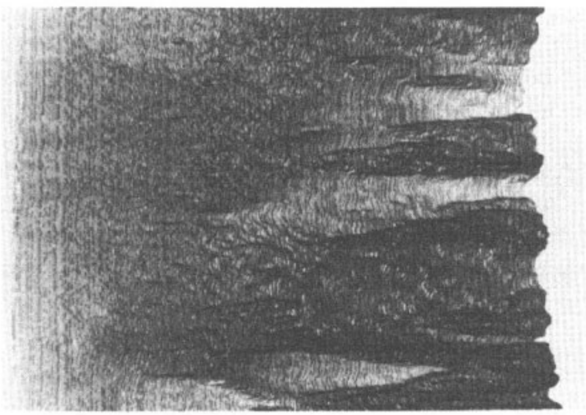

Figure 1. Plotting the digitized image of the meniscus at different times. The shift between the patterns is proportional to the time lapse between the pictures. The three-dimensional appearance of the patterns is obtained by making the brightness of the surface at a given point proportional to its height.

The dynamic scaling of the growing interface is demonstrated by the data displayed in figure 2, where the logarithm of the square root of the height–height correlation function for $x = 0$ is plotted against ln t. The slope of the straight line fitted to the data shows that the roughening process is characterized by the exponent $\beta \simeq 0.65$. We note that this exponent had not been previously measured experimentally in any system and its value is significantly higher than that predicted by various models [5–12].

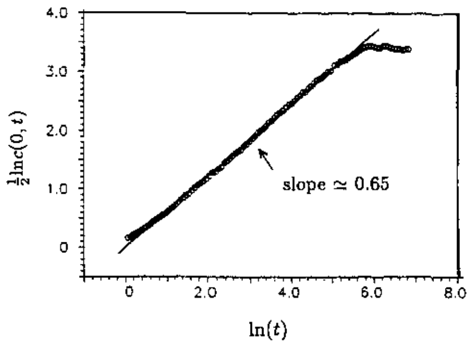

Figure 2. Height-height correlation function for $x=0$ is plotted against $\ln t$. The slope of the straight line fitted to the data indicates that the roughening exponent $\beta \simeq 0.65$.

The spatial scaling of the correlations is shown in figure 3 where the logarithm of the square root of the correlation function is plotted against $\ln x$. The slope of the straight line fitting the initial part of the date set is about $\alpha \simeq 0.81$, while the long wavelength behaviour is described by the exponent $\alpha \simeq 0.49$.

Our results demonstrate that the evolution of the interface in the experiments on two fluid displacement in porous media is governed by dynamic scaling. The values of the surface exponents α and β appear to be different from the corresponding values obtained in various models of interface growth [5-14]. On the other hand, the fact that our values of α and β differ from the values reported for models of two-dimensional marginally stable growth [5-12] is consistent with other experiments [19, 20, 23], where the static exponent has been found to be different from 1/2. Furthermore, our values are in a reasonable agreement with the scaling law [25, 26] $\alpha + \alpha/\beta = 2$, since $0.81 + 0.81/0.65 \simeq 2.06$.

The non-universality of our values is most likely due to the fact that the fluctuations affecting the surface growth (introduced by the random distribution of throats between the beads) cannot be described in terms of a white, Gaussian, uncorrelated noise as is assumed in the original paper on the continuum equation approach [12]. One

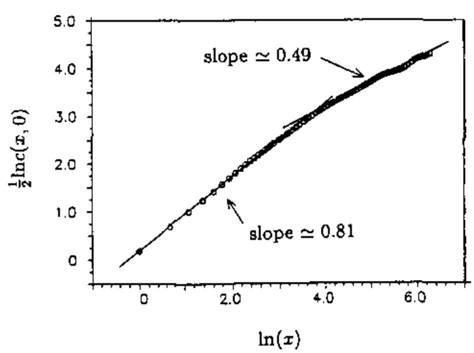

Figure 3. Correlations in the fluid displacement experiment. The logarithm of the correlation function is plotted against $\ln x$. The slope of the straight line fitting the initial part of the data set is about $a \simeq 0.81$, while the long wavelength behaviour is described by the exponent $\alpha \simeq 0.49$.

possibility is that there exists temporal or spatial correlation in the noise which has been shown to lead to non-universality by Medina *et al* [26]. However, there have been only a few numerical studies of the effects of noise correlations in model systems [27] consistent with the above theoretical prediction.

Very recently Zhang proposed a more physically motivated approach [28] assuming a local noise which is distributed according to a power law instead of following a Gaussian distribution. The related simulations [28, 29] and theoretical considerations [29, 30] are consistent with our values for the exponents. Finally, a recent direct numerical simulation of the advancing interface of a wetting fluid in a random array of hard disks [31] resulted in a static exponent α close to the experimental value reported in this letter. Further experimental and theoretical investigations are expected to provide additional relevant information on the origin of the non-universal exponents observed in the experiments.

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